Rational Numbers---Adding Fractions With Like Denominators.

A. <u>In Words:</u> To **add** fractions with ***like*** denominators, add the numerators and write the ***sum*** over the same denominator.

B. <u>In Symbols:</u> For fractions $\frac{a}{c}$ and $\frac{b}{c}$, where $c \neq 0$, $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$.

C. <u>Example:</u> $\left(\frac{2}{5}\right) + \left(\frac{1}{5}\right)$ becomes $\frac{2+1}{5}$ which produces the result $\frac{3}{5}$.

Rational Numbers---Subtracting Fractions With Like Denominators.

A. <u>In Words:</u> To subtract fractions with like denominators, subtract the numerators and write the ***difference*** over the ***same denominator***.

B. <u>In Symbols:</u> For fractions $\frac{a}{c}$ and $\frac{b}{c}$, where $c \neq 0$, $\frac{a}{c} - \frac{b}{c} = \frac{(a-b)}{c}$.

1. <u>Example:</u> $\left(\frac{2}{5}\right) - \left(\frac{1}{5}\right)$ becomes $\frac{2-1}{5}$ which produces the result $\frac{1}{5}$.

Rational Numbers---Adding and Subtracting Unlike Fractions (When the Denominators are NOT the Same).

A. To add or subtract fractions with unlike denominators:

1. Find the Least Common Denominator (LCD).
2. Rename the fractions with a common denominator.
3. Add or subtract the numerators.
4. Place the sum or difference over the common denominator.
5. Simplify.

Rational Numbers---The Least Common Denominator (LCD) Defined.

A. You find the LCD between two or more fractions by finding the Least Common Multiple (LCM) of the denominators of the fractions.

B. The Least Common Multiple.

1. A ***multiple*** of a number is a ***product*** of that number and any ***whole*** number.

2. **Multiples** that are **shared** by two or more numbers are called **common multiples**.

3. The **least** of the common **non-zero** multiples of two or more numbers is called the **Least Common Multiple (LCM)**.

Rational Numbers---Finding The Least Common Denominator (LCD).

A. ***You can find the LCM by making a chart:***

1. Example: Find the Least Common Denominator for 1, 2, 3, & 6

x	1*x	2*x	3*x	4*x	5*x	6*x
0	0	0	0	0	0	0
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	**6**	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

The LCM of 1, 2, 3, & 6 is 6.

I. IMPORTANT REVIEW: Prime Numbers

A. A **prime number** is a whole number greater than one that has **exactly two factors**, 1 and itself.

B. Prime Factorization.

1. When a positive integer (other than one) is expressed as ***a product of factors that are all prime***, the expression is called the **prime factorization**.

Rational Numbers---Finding The Least Common Denominator (LCD).

A. ***You can find the LCM by finding the Prime Factorization of each Number*.**

1. Find the common denominators and add: $\frac{3}{8} + \frac{5}{12}$

a. Find the prime factors of each denominator. Express these factors as powers.

Number	Prime Factors	Powers
8 =	$2 \cdot 2 \cdot 2$	2^3
12 =	$2 \cdot 2 \cdot 3$	$2^2 \cdot 3$

b.. List all the powers in increasing order of their exponents, using each **only once.**

$2^2 \cdot 2^3 \cdot 3 = 96$ The Least Common Multiple (LCD) will be 96.

c. Multiply the numerator and the denominator by the factor needed to change the denominator into the LCD (96)

1. For the fraction $\frac{3}{8}$ find how many times 8 goes into 96. Multiply this number by both the numerator and the denominator:

a. $\frac{96}{8} = 12$ so $\frac{3 \cdot 12}{8 \cdot 12} = \frac{36}{96}$

2. For the fraction $\frac{5}{12}$ find how many times 12 goes into 96. Multiply this number by both the numerator and the denominator:

a. $\frac{96}{12} = 8$ so $\frac{5 \cdot 8}{12 \cdot 8} = \frac{40}{96}$

2. So $\frac{3}{8} + \frac{5}{12} = \frac{36}{96} + \frac{40}{96} = \frac{36+40}{96} = \frac{76}{96} = \frac{19}{24}$

1. A short-cut way to find a common denominator is to ***multiply the numerator & denominator of the first fraction by the denominator of the second fraction***. ***Then multiply the numerator & denominator of the second fraction by the denominator of first fraction.***

a. Example $\left(\frac{2}{5}\right) + \left(\frac{1}{11}\right)$ becomes $\left(\frac{11 \cdot 2}{11 \cdot 5}\right) + \left(\frac{5 \cdot 1}{5 \cdot 11}\right)$

which becomes $\left(\frac{22}{55}\right) + \left(\frac{5}{55}\right)$ which $\left(\frac{22+5}{55}\right)$ becomes $\left(\frac{27}{55}\right)$

1. Divisibility Rules:

A number is divisible by:

a. 2 if the ones digit is divisible by 2
b. 3 if the sum of its digits is divisible by 3
c. 4 if the last two digits are divisible by 4
d. 5 if the ones digit is 0 or 5
e. 6 if the number is divisible by 2 and 3
f. 8 if the last three digits are divisible by 8
g. 9 if the sum of all the digits is divisible by 9
h. 10 if the ones digit is 0

Simplifying Fractions---Using Prime Factors.

A. A fraction can be simplified two ways:

1. Break each number into it's prime factors.

a. Example 1: $\frac{6}{9}$ becomes $\frac{2 \cdot 3}{3 \cdot 3}$.

1. Next, cancel out the common factors.

a. We cancel out the 3, leaving only 2 in the numerator and a 3 in the denominator.

b. The remaining fraction is in it's simplest form: $\frac{2}{3}$

Simplifying Fractions---Using the Greatest Common Factor .

A. A fraction can be simplified two ways:

1. **Break each number into it's prime factor** (as in XI above)

2. **Using the Greatest Common Factor**.

3. Example: Write $\frac{12}{40}$ in simplest form.

   a. Find the prime factorization of the numerator 12 and the denominator 40.

      1. $\frac{12}{40} = \frac{2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 9}$

      2. $\frac{2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 9}$ has two factors in common, $2 \cdot 2$.

      3. The product $(2 \cdot 2) \Rightarrow 4$, ***is the greatest common factor*** of the numerator and the denominator.

      4. By canceling out the common factors, you are in affect ***dividing both the numerator and denominator by the greatest common factor***.

   b. The product of the remaining factors, $\frac{3}{2 \cdot 5} \Rightarrow \frac{3}{10}$, is the simplified form of the fractional rational number $\frac{12}{40}$.